Review of Evidence-based Mathematics Interventions for Students with Autism Spectrum Disorders

Juliet E. Hart Barnett and Shannon Cleary
Arizona State University

Abstract: Students with autism spectrum disorders (ASD) are being included more frequently in the general educational setting, and are therefore increasingly expected to access and master core curricular content, including mathematics. However, mathematics often presents challenges to students with ASD. Interventions to improve the mathematics skills of students with ASD have been recommended. This comprehensive literature review synthesized eleven studies of mathematics intervention strategies for students with ASD. Though studies related to instructional interventions in mathematics for students with ASD are limited, these students can benefit from mathematics interventions, which can help them strengthen their mathematics skills, increase independence when completing problems, and use acquired skills in community or other applied settings. Future implications include the need for additional, empirically-supported interventions in mathematics for students with ASD and the need to target more academically-oriented math interventions for this population, particularly in the context of problem solving, which will assist in determining the potential of students with ASD to achieve mathematical success.

The education of students with autism spectrum disorders (ASD) in inclusive classroom environments is becoming increasingly common (Cihak, Fahrenkrog, Ayres, & Smith, 2010). Placement of students on the spectrum in general education has increased more quickly than all other disability categories combined (Sansoti & Powell-Smith, 2008). Today, 36 percent of students with ASD spend more than 80 percent of their school day in general education classes, which constitutes a significant increase from their 4.8 percent inclusion rate in 1991 (Whitby, 2013). As a result, although students’ individual IEP goals and needs determine special education services, there is a rising expectation that these learners will access and master the same core curricular content as their typically developing peers and ushers in a related demand for effective educational interventions to promote students’ successful content acquisition (Knight, Smith, Spooner, & Browder, 2012).

However, instructional programs for students with ASD tend to focus on communication and social skills (Plavnik & Ferreri, 2011; Wang & Spillane, 2009) as well as functional and life skills as opposed to traditional age and grade level content areas (Cihak & Grim, 2008; Rayner, 2011). Within the research base related to academic content, the focus for students with ASD is primarily related to reading (Bouck, Satsangi, Taber-Doughty, & Courtney, 2014; Delano, 2007). Although mathematics education is a national priority for all students (Ellis & Berry, 2005), there are fewer comparable, empirically supported interventions in mathematics than in literacy for students with ASD (Bouck et al., 2014).

Mathematics is an area of academic concern for students with ASD. Nearly 25 percent of students with ASD contend with a mathematics learning disability (Mayes & Calhoun, 2006). A recent longitudinal study of individuals with ASD indicated slower growth rates in calculation skills as compared to students with learning disabilities (Wei, Lenz, & Blackorby, 2013). In addition, although many of these learners maintain adequate mathematics performance in the earlier grades when rote memorization of facts and procedures is im-
portant (Chiang & Lin, 2007), the same students may struggle as they enter middle school when the content becomes more abstract, cognitively complex, and emphasizes problem solving, higher level thinking, and mathematical reasoning, which are cited by researchers as areas of weakness for children with ASD (Mayes & Calhoun, 2003; Whitby & Mancil, 2009).

The difficulties students with ASD encounter in mathematics may likely stem from deficits in executive functioning, including planning, organization, working memory, mental flexibility, attention, self-monitoring, and impulse control (Happe, Booth, Charlton, & Hughes, 2006; Hughes, Russell, & Robbins, 1994). Additionally, the language impairment associated with ASD may also negatively impact mathematics development across several areas such as number-word sequence, calculation, fact retrieval, and in particular, problem-solving (Donlan, 2007; Zentall, 2007), which requires students to utilize both semantic and numeric information (Rockwell, Griffin, & Jones, 2011). Despite the fact that researchers have investigated the role of these student-level characteristics on mathematical skills, they also recognize the difficulty in extrapolating the extent to which these challenges impact math performance given the limited literature base (Chiang & Lin, 2007; Fleury et al., 2014), which could also suggest that ASD students’ difficulties with mathematics may be related to a lack of targeted direct instructional opportunities in this area. Notwithstanding, the general education curriculum and state assessments in mathematics increasingly emphasize the importance of developing the conceptual understanding and problem solving described and recommended by the National Council for Teachers of Mathematics (NCTM) across domain areas (Botte, 2001; NCTM, 2002; Rockwell et al., 2013), which students with ASD who are educated in inclusive settings will likely encounter.

Given the rise of students with ASD in inclusive settings, their documented difficulties in mathematical understanding and problem solving, and the prominence the NCTM has placed on conceptual understanding and problem solving across skill areas, examining and evaluating the extant research in the area of mathematics interventions for learners with ASD is a national research priority. The purpose of this review is to contribute to the current knowledge base on effective mathematics interventions for students with ASD in order to ensure that their teachers consider and implement effective teaching techniques to help their students acquire the mathematics skills identified on their IEPs and those being taught in the general education classroom, and achieve maximum success.

Method

To be included in this comprehensive review, published articles had to meet the following criteria: (a) be published in a peer-reviewed journal, (b) include students of any age ranging from elementary age through post secondary education, (c) include participants identified with an ASD diagnosis (i.e., autism, PDD-NOS, or Asperger syndrome), (d) be conducted in any educational or outside clinical, home-based, or tutorial setting (e.g., general education classroom, special education resource or self-contained classroom, etc.), and (e) describe research studies evaluating the effectiveness of academic or functional interventions targeting mathematical content standards (i.e., number and operations, algebra, geometry, measurement, and data analysis/probability) and/or process standards (i.e., problem-solving, reasoning, communication, connections, and representations) as described by the NCTM.

During the review process, exclusionary criteria were also developed. First, studies lacking an empirical research design were excluded. That is, articles featuring mathematics teaching strategies or tips for students with ASD were eliminated to include only articles featuring an administered intervention strategy supported with data/evidence to determine the effectiveness of that intervention strategy. Second, articles that featured mathematics problems as a component of a larger investigation of a teaching strategy, but did not use the study of mathematics interventions as the primary goal of the experiment, were excluded from the literature review.

The search was conducted electronically using online ERIC and Academic Search Premier databases with no limits placed on year of publication. Key search terms pertained to autism and mathematics (e.g., autism spec-
trum disorder, Asperger’s Syndrome, autism, intellectual disabilities, mathematics, mathematics instruction, teaching strategies, interventions, etc.) From this initial search, journals typically publishing intervention research in ASD were identified: *Journal of Autism and Developmental Disorders, Focus on Autism and Other Developmental Disabilities, Research in Autism Spectrum Disorders, Behavioral Interventions, Preventing School Failure*, and *Education and Training in Autism and Developmental Disabilities.* Furthermore, multiple mathematics journals were also searched for any additional articles that met the search criteria. Mathematics education journals included in the search were: *Journal for Research in Mathematics Education, Journal of Mathematics Teacher Education, Educational Studies in Mathematics,* and *Mathematics Education Research Journal.* These journals were electronically hand searched for all abstracts of interventions related to improving the mathematics knowledge or skills of learners with ASD.

To ensure reliability of the search process, the two authors of the study and a graduate student independently conducted the initial review. Reliability was then calculated by dividing the number of agreements by the number of agreements plus disagreements and multiplying by 100. Inter-observer agreement (IOA) was 85%. Through closer examination of all studies initially located as well as discussion of inclusionary and exclusionary criteria, 100% IOA was achieved through consensus. In sum, the search was conducted to obtain and review all available research related to mathematics education interventions that have been conducted with students with ASD and that had as their primary goal increasing these students’ mathematical skills in alignment with NCTM standards.

### Results

The initial search produced 13 studies, of which, two were eliminated based on failure to meet inclusionary criteria. Therefore, eleven studies met the aforementioned inclusionary criteria. Table 1 shows the participants, setting, intervention, intervention testing, design, dependent variables, independent variables, and treatment effects/results of each study reviewed. Studies were largely single-subject design, were grouped by similarity of intervention type, and were designated as an intervention implementing either visual representations or some form of cognitive strategy instruction to target specific mathematics skills. In addition, studies will be described in terms intervention setting (i.e., self-contained vs. general education classroom) as well as the mathematics skill targeted (i.e., academic vs. functional skills). Lastly, implications for practitioners and directions for future research will also be discussed.

Of the 11 studies, six implemented visual representations to develop specific math skills (Bouck et al., 2014; Burton, Anderson, Prater, & Dyches, 2013; Cihak & Foust, 2008; Fletcher, Boon, & Cihak, 2010; Rockwell et al., 2011; Waters & Boon, 2011). Visual representations include manipulatives, pictures, number lines, and graphs of abstract concepts, functions, and relationships (Xin & Jitendra, 1999). Representation approaches to solving mathematical problems include pictorial (e.g., diagramming), concrete (e.g., manipulatives, or objects that assist students in understanding abstract mathematical concepts or properties by representing them in multiple ways), verbal (linguistic training), and mapping instruction (schema-based) (Xin & Jitendra, 1999). The 5 remaining studies implemented some form of cognitive or meta-cognitive strategy instruction (Banda & Kubina Jr., 2010; Cihak & Grim, 2008; Hua, Morgan, Kaldenberg, & Goo, 2012; Rapp et al., 2012; Whitby, 2013). Cognitive strategies are frequently used in mathematics problem solving and are defined as a series of sequenced procedures that permit a student to complete a task effectively using rules, processes, and steps that are applied systematically to obtain a problem solution (Simpson, 2005), and include the metacognitive elements of when and where to apply specific strategies in the implementation and evaluation of the process and outcome (Montague, 2008; Reid & Leinemann, 2006). Both of these approaches are among those considered evidence-based for improving mathematical knowledge and skills of students with disabilities, particularly those who are low achieving and those with learning disabilities (Baker, Gersten, & Lee, 2002), and

---

<table>
<thead>
<tr>
<th>Author(s)</th>
<th>Participants in Experiment (Total Number of Participants, Participants with ASD, Ages)</th>
<th>Setting</th>
<th>Intervention</th>
<th>Intervention Testing</th>
<th>Design</th>
<th>Dependent Variables</th>
<th>Independent Variables</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bouck, Satsangi, Dougherty, Courtney (2013)</td>
<td>N = 3 (N_{ASD} = 3), Ages = 6, 7, 10</td>
<td>Autism Clinic</td>
<td>Using concrete (physical objects) and virtual (3D objects on computer) manipulatives to help students acquire single-digit and double-digit subtraction skills</td>
<td>Virtual or concrete manipulative sessions consisting of five subtraction problems each</td>
<td>Single Subject Alternating Treatment Design</td>
<td>Percentage of accurately completed subtraction problems and percentage of subtraction problem steps completed individually</td>
<td>Use of concrete and virtual manipulatives</td>
<td>All three participants demonstrated an increase in correctly completed subtraction problems using both the concrete and virtual manipulatives (the virtual manipulatives proved slightly more effective), as well as an increase in independent completion of problems</td>
</tr>
<tr>
<td>Burton, Anderson, Prater, and Dyches (2013)</td>
<td>N = 4 (N_{ASD} = 3), Ages = 13, 13, 14, 15</td>
<td>Junior High School</td>
<td>Using a video self-modeling technique to teach money estimation of a given item and estimation of the amount to receive in change</td>
<td>Two daily sessions of intervention testing to complete five math problems with the iPad, occurring four days each week</td>
<td>Multiple-Baseline Across-Participants Design</td>
<td>Percentage of accurately completed money computational problems</td>
<td>Use of video self-modeling on the iPad</td>
<td>All four of the participants demonstrated an improvement in math skill performance after implementation of the video self-monitoring</td>
</tr>
<tr>
<td>Cihak and Foust (2008)</td>
<td>N = 3 (N_{ASD} = 3), Ages = 7, 7, 8</td>
<td>Resource room at Elementary School</td>
<td>Using number line and touch-point strategies to solve single-digit addition math problems</td>
<td>Two daily 5-20 minute sessions to complete a worksheet of ten single-digit mathematics problems using either touch points or a number line</td>
<td>Alternating Treatments Design</td>
<td>Percentage of correct single-digit addition math problems completed</td>
<td>Use of a touch points and use of a number line</td>
<td>All three students demonstrated that touch-point strategy was more successful in teaching single-digit addition skills than the number line strategy</td>
</tr>
<tr>
<td>Fletcher, Boon, and Chihak (2010)</td>
<td>N = 3 (N_{ASD} = 2), Ages = 13, 13, 14</td>
<td>Self-contained classroom at Middle School</td>
<td>Teaching single-digit mathematics problems using TOUCHMATH, a multi-sensory mathematics program, and a number line</td>
<td>Two daily 5-15 minute sessions to complete a worksheet of ten single digit mathematics problems using either the “touch points” or number line strategy</td>
<td>Alternating Treatments Design</td>
<td>Percentage of single-digit mathematics problems answered correctly</td>
<td>Use of the TOUCHMATH program using “touch points” and the number line strategy</td>
<td>TOUCHMATH strategy was more effective and efficient in teaching single-digit addition problems compared to the number line strategy</td>
</tr>
<tr>
<td>Rockwell, Griffin, and Chihak (2011)</td>
<td>N = 1 (N_{ASD} = 1), Age = 10</td>
<td>Author’s Home Office</td>
<td>Using schematic diagrams to solve group, change, and compare addition or subtraction word problems</td>
<td>Problem solving probe sessions containing two group, two change, and two compare word problems</td>
<td>Multiple Probes Across Behaviors Single-Case Design</td>
<td>Percentage of word problems completed accurately</td>
<td>Use of schematic diagrams to solve group, change, and compare word problems</td>
<td>The participant successfully increased her ability to complete single-step addition and subtraction word problems</td>
</tr>
<tr>
<td>Author(s)</td>
<td>Participants in Experiment (Total Number of Participants, Number of Participants with ASD, Ages)</td>
<td>Setting</td>
<td>Intervention</td>
<td>Intervention Testing</td>
<td>Design</td>
<td>Dependent Variables</td>
<td>Independent Variables</td>
<td>Results</td>
</tr>
<tr>
<td>-----------------------------------</td>
<td>------------------------------------------------------------------------------------------------</td>
<td>----------------------------------------------------------------------------------------------------------------</td>
<td>-----------------------------------------------------------------------------------------------------------</td>
<td>----------------------------------------------------------------------------------------------------------------</td>
<td>------------------------------------------</td>
<td>---------------------</td>
<td>----------------------</td>
<td>-----------------------------------------------------------------------------------------------</td>
</tr>
<tr>
<td>Waters and Boon (2011)</td>
<td>N = 3 N(ASD) = 1 N(A) = 1 Ages = 14, 15, 16</td>
<td>Self-contained special education mathematics classroom at Public High School</td>
<td>Teaching 3-digit money computation subtraction problems with regrouping using the TouchMath program</td>
<td>Ten 3-digit money computation problems solved by regrouping using the TouchMath program</td>
<td>Multiple-Probe Across Participants Design</td>
<td>Percentage of correct 3-digit money computations performed</td>
<td>Use of TouchMath program using “touch points” and regrouping</td>
<td>All three participants increased acquired skills to subtract 3-digit mathematics operations using money computations</td>
</tr>
<tr>
<td>Banda and Kubina Jr. (2010)</td>
<td>N = 1 N(ASD) = 1 Age = 13</td>
<td>Resource room at Middle School</td>
<td>Using high preference math tasks to increase academic compliance and completion of low-preference math tasks</td>
<td>Ten cards containing two high-preference problems to be completed prior to one low-preference problem</td>
<td>ABAB Design</td>
<td>Lack of initiation of a three-digit by three-digit missing addend problem</td>
<td>Using a high preference intervention to complete ten test cards containing two three-digit by three-digit addition problems followed by one missing addend problem</td>
<td>The student took less time to begin low-preference math problems that were stated after high-preference math problems</td>
</tr>
<tr>
<td>Cihak and Grim (2008)</td>
<td>N = 4 N(ASD) = 4 Ages = 15, 16, 16, 17</td>
<td>Resource room at High School, school bookstore, and local department store</td>
<td>Using counting-on math technique with the next-dollar strategy to increase independent purchasing skills</td>
<td>1) Classroom Phase: Two daily sessions containing ten trials of ten problems to be completed using the counting-on strategy 2) Bookstore and Community Phases: One daily session containing three purchasing trials</td>
<td>Multiple-Probe Design</td>
<td>Percentage of independent purchases completed accurately</td>
<td>Use of counting-on and next-dollar math strategies to enhance purchasing skills</td>
<td>All four students successfully learned the counting-on and next-dollar techniques and were able to apply these skills to community settings</td>
</tr>
<tr>
<td>Hua, Morgan, Kaldenburg, and Goo (2012)</td>
<td>N = 5 N(ASD) = 3 N(A) = 2 Ages = 18, 18, 21, 22, 22</td>
<td>University classroom</td>
<td>Using a three-step cognitive strategy (TIP) for calculating tip and total bill for young adults with intellectual disabilities</td>
<td>Six instructional stages including a final stage of ten tip and total bill problems to be completed independently by the students</td>
<td>Pre- and Posttest Nonequivalent Groups Design</td>
<td>Total number of tip and total bill calculations completed accurately</td>
<td>Use of TIP method</td>
<td>The experimental group successfully increased their ability to calculate tip and total bill using the TIP strategy, showing much higher posttest result than the comparison group</td>
</tr>
<tr>
<td>Author(s)</td>
<td>Participants in Experiment (Total Number of Participants / Number of Participants with ASD, Ages)</td>
<td>Setting</td>
<td>Intervention</td>
<td>Intervention Testing</td>
<td>Design</td>
<td>Dependent Variables</td>
<td>Independent Variables</td>
<td>Results</td>
</tr>
<tr>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
<td>---</td>
</tr>
<tr>
<td>Rapp, Marvin, Nystedt, Swanson, Paananen, and Tabatt (2012)</td>
<td>$N = 4$ $N(ASD) = 2$ $N(MMR) = 1$ Ages = 7, 8, 9, 12</td>
<td>Individual classroom at School or at Home</td>
<td>Using response repetition as an error-correction technique to increase students’ ability to complete math facts and math computation</td>
<td>Two to four weekly sessions ranging from 15–20 minutes each, containing 2–5 sets of flash cards or worksheets per session</td>
<td>Nonconcurrent and Concurrent Multiple Baseline Designs</td>
<td>Percentage of math problems solved accurately</td>
<td>Use of Response Repetition technique</td>
<td>Three of four participants demonstrated improvement on targeted math problems, and two of two participants showed improvement on other forms of math problems</td>
</tr>
<tr>
<td>Whitby (2013)</td>
<td>$N = 3$ Ages = 7th grader, 7th grader, 8th grader</td>
<td>Individual Public Middle School classrooms</td>
<td>Teaching math word problems using the 7 cognitive strategies and 3 metacognitive strategies from the Solve It! Problem Solving Routine</td>
<td>Minimum of five training sessions containing 3–5 math word problems; five acquisition condition sessions containing five math word problems</td>
<td>Multiple Baseline Across Participants Design</td>
<td>Percentage of correct word problems completed</td>
<td>Use of Solve It! Problem Solving Routine curriculum</td>
<td>All three participants learned to use problem-solving skills to accurately complete math word problems, as demonstrated through the increased percentage of correct word problem responses</td>
</tr>
</tbody>
</table>

**Note:** Table 1 lists the reviewed articles alphabetically based on Visual Representations or Cognitive Strategy Intervention type

* Excludes comparison group participants
therefore may have useful application to students with ASD.

In all, 34 students comprised the participants in the studies included in the review, with 28 participants diagnosed with an ASD (82.35%). The participants with ASD included three students diagnosed with Asperger’s Syndrome (8.82%), one student diagnosed with Pervasive Development Disorder (2.94%), and 17 students diagnosed with Autism (50.00%). Participants ranged from 6 to 22 years, and included students attending elementary school through individuals attending post-secondary programs for young adults with disabilities. A summary of participant characteristics is provided in Table 2. Moreover, although no specific date restrictions were utilized in the search, studies located in the review were published between 2008 and 2013. Specifically, there were three studies from 2012, two from each of the remaining years, and no studies from 2009.

The six studies incorporating visual representation approaches consisted of three interventions using touch point’ (Cihak & Foust, 2008; Fletcher et al., 2010; Waters & Boon, 2011), one intervention using video self-modeling on an iPad (Burton et al., 2013), one intervention using virtual and concrete manipulatives (Bouck et al., 2014), and one intervention using schematic diagrams to solve word problems (Rockwell et al., 2011).

Of the visual representation interventions reviewed, three studies used the “touch-point” technique to teach mathematic skills to students with ASD (Cihak & Foust, 2008; Fletcher et al., 2010; Waters & Boon, 2011). Two of these interventions investigated the use of touch-points in comparison to a number line when teaching single-digit mathematics problems. During the first of these two studies (Cihak & Foust, 2008), instruction was provided to three students with autism. Students were presented a number line labeled with numbers zero through twenty and taught how to use the tool by moving their fingers along the line in correspondence to the numbers in each problem. Touch point instruction included teaching students the dot positions for the numbers one through nine, instructing students to count out loud the dots of the two given numbers, and writing the final number said aloud. Daily intervention sessions consisted of students performing a worksheet with ten single-digit math problems, alternating between using the touch point and number line methods. Results showed that the touch point method was more successful than the number line, and participants averaged 72% of the problems solved correctly using touch point, while the number line strategy averaged 17% of correctly solved problems (Cihak & Foust, 2008).

During the second number line and touch point intervention (Fletcher et al., 2010), three students with moderate intellectual disabilities and two diagnosed with ASD, were taught single-digit mathematics problems. The students were taught the same procedures to utilize the number line and touch point methods as was used in the previous intervention. Each intervention session required participants to solve ten single-digit

<p>| TABLE 2 |
| Participant Characteristics |</p>
<table>
<thead>
<tr>
<th>Student Diagnosis</th>
<th>Number of Students with Diagnosis</th>
</tr>
</thead>
<tbody>
<tr>
<td>Severe LD/ADHD</td>
<td>1</td>
</tr>
<tr>
<td>ID</td>
<td>1</td>
</tr>
<tr>
<td>ASD and ID</td>
<td>2</td>
</tr>
<tr>
<td>ASD</td>
<td>26</td>
</tr>
<tr>
<td>-Autism</td>
<td>9</td>
</tr>
<tr>
<td>-Autism and MID</td>
<td>7</td>
</tr>
<tr>
<td>-Autism and PDD</td>
<td>1</td>
</tr>
<tr>
<td>-AS</td>
<td>1</td>
</tr>
<tr>
<td>-AS and ID</td>
<td>1</td>
</tr>
<tr>
<td>-AS and MID</td>
<td>1</td>
</tr>
<tr>
<td>-ASD</td>
<td>6</td>
</tr>
<tr>
<td>MID</td>
<td>2</td>
</tr>
<tr>
<td>MR</td>
<td>1</td>
</tr>
<tr>
<td>MR and ID</td>
<td>1</td>
</tr>
<tr>
<td><strong>Total Number of Participants</strong></td>
<td><strong>34</strong></td>
</tr>
</tbody>
</table>

Table Abbreviations:
LD = Learning Disabilities
ADHD = Attention Deficit Hyperactivity Disorder
ID = Intellectual Disabilities
ASD = Autism Spectrum Disorder
MID = Mild Intellectual Disabilities
MR = Mental Retardation
PDD = Pervasive Development Disorder
AS = Asperger Syndrome

Interventions Using Visual Representations
addition problems using either the number line or touch point method. Consistent with the prior study that implemented these two strategies, the touch point method proved to be more effective in teaching single-digit math problems. Results show that participants solved an average of 92% of the problems correctly using the touch points, whereas they averaged 30% of problems using the number line (Fletcher et al., 2010).

Waters and Boon (2011) provided instruction to three students with mild intellectual disabilities, with two having an additional diagnosis of ASD. This third visual representation study used the touch-point strategy to teach three-digit money computational subtraction problems with regrouping. After mastering the TouchMath program’s procedure for using touch points, students were taught the method for solving subtraction problems with regrouping. This method follows a similar procedure to the addition touch-point strategy, but requires students to also count backwards and borrow numbers when regrouping. Results suggest that the touch point strategy is effective as demonstrated by each student’s increase in ability to solve three-digit subtraction problems with regrouping, a skill that each student lacked prior to the intervention (Waters & Boon, 2011).

Burton et al. (2013) provided training to three students with autism and one student diagnosed with an intellectual disability. Researchers implemented video self-modeling to teach money estimation of a given item, as well as the estimated amount to receive in change for that item. Prior to the intervention, a video was recorded of each student, in which the participant used a script featuring the seven steps of task analysis to solve a math problem. This procedure was repeated five times, resulting in five recorded videos. During the intervention, the student watched the video to observe himself complete the problem, and was able to pause, fast-forward, or rewind the recording as he completed the same problem on paper. The observed functional relationship between video self-modeling and performance supports the idea that this study was successful in improving the money skills of students with ASD (Burton et al., 2013).

Rockwell et al. (2011) implemented an additional visual representation intervention using schematic diagrams to help a student with ASD solve group (featuring two smaller parts combined to form one larger group), change (featuring a beginning amount, change amount signified by an action, and an ending amount), and compare (featuring a larger amount, smaller amount, and a difference) addition or subtraction word problems. The increased percentage of correct word problem responses in this study suggested that this intervention was effective (Rockwell et al., 2011).

A final intervention that utilized virtual and concrete manipulatives compared the effectiveness of these two strategies when implemented by students with ASD. Bouck et al. (2014) implemented this intervention with three students with ASD. Intervention trials alternated between students using either concrete blocks or an online computer program to solve subtraction problems. Although students increased the number of successfully completed single- and double-digit math problems using both strategies, the virtual manipulatives technique proved slightly more effective (Bouck et al., 2014).

Overall, visual representations were effective strategies in teaching students with ASD mathematic skills. Authors reported gains in single-digit addition problems (Cihak & Foust, 2008; Fletcher et al., 2010), subtraction problems (Bouck et al., 2014; Waters & Boon, 2011), money estimation (Burton et al., 2013), and addition/subtraction word problems (Rockwell et al., 2011). However, a limitation to the studies may include the fact that the majority of the visual representation interventions (Bouck et al., 2014; Burton et al., 2013; Rockwell et al., 2011; Waters & Boon, 2011) featured participants who received mathematic instruction outside of the general education setting, likely preventing them from accessing the mathematic content taught according to general education standards.

Interventions Using Cognitive Strategy Instruction

The five studies implementing cognitive strategies included two interventions citing specific use of cognitive or meta-cognitive strategies (Hua et al., 2012; Whitby, 2013), one intervention using response-repetition as an error-correction procedure (Rapp et al.,
2012), one intervention using counting-on and next-dollar strategies (Cihak & Grim, 2008), and one intervention using a high-preference strategy to increase completion of low-preference tasks (Banda & Kubina Jr., 2010).

Two of the reviewed interventions used cognitive strategies to aid students in memorizing the steps required to successfully complete assigned mathematics problems (Hua et al., 2012; Whitby, 2013). Whitby (2013), the first of these interventions, achieved the goal of teaching students to accurately complete mathematic word problems by instructing students to memorize and apply Solve It! Problem Solving Routine’s seven cognitive steps (i.e., read-paraphrase-visualize-hypothesize-estimate-compute-check). This study also required the three participants with ASD to utilize the program’s three meta-cognitive strategies, which included self-management, self-questioning, and self-evaluation. During the intervention, all three students solved more problems accurately than their peers, suggesting the Solve It! method to be an effective strategy for teaching problem solving skills (Whitby, 2013).

The second intervention (Hua et al., 2012) taught five students with either ASD or intellectual disability to utilize a three-step cognitive strategy, TIP, to calculate tip and total bill. Results of this cognitive strategy-based intervention showed the method was effective in helping students complete tip and total bill computations, as well as in aiding students to accurately apply word problem solving techniques. Additionally, these results confirm that cognitive strategies are beneficial in helping students with ASD memorize processes required to successfully solve mathematic problems (Hua et al., 2012).

Rapp et al. (2012) utilized a cognitive strategy intervention of response repetition as an error-correction technique to increase computation of math facts. The study included four students diagnosed with disabilities, with two students with ASD, one with Asperger syndrome, and one with a moderate intellectual disability. During the intervention phase, a trainer presented math problems and provided verbal praise for accurate answers. If the student provided an incorrect answer, the trainer prompted the student to verbally state the correct answer five times. Response repetition was demonstrated to be a successful cognitive strategy for three of four participants (Rapp et al., 2012).

Cihak and Grim (2008) studied four students with moderate intellectual disabilities and ASD. Students were taught counting-on and next-dollar math techniques to increase independent purchasing skills. With the next-dollar strategy, students were taught to pay with one more dollar than asked for by the sales associate. The counting-on method taught students to count on from the first addend to obtain the sum. Results of this cognitive strategy showed that students developed and maintained independent purchasing skills, and were able to apply these skills to community settings (Cihak & Grim, 2008).

The final cognitive strategy study (Banda & Kubina Jr., 2010) required a middle school student with ASD to complete high-preference mathematics problems followed by the completion of low-preference problems, with the goal of increasing the initiation of the low-preference problems. Each intervention phase featured a stack of ten cards containing two high-preference problems followed by one low-preference problem for the student to complete. The results of this intervention showed that the student not only took less time to begin low-preference math problems, but also increased performance on the low-preference three-digit by three-digit missing addend problems (Banda & Kubina Jr., 2010).

In summary, cognitive strategy interventions were successful in increasing students’ mathematics skills and understanding. Authors discussed improvements among students with ASD in solving word problems (Whitby, 2013), money computational problems (Cihak & Grim, 2008; Hua et al., 2012), three-digit by three-digit missing addend problems (Banda & Kubina Jr., 2010), and addition, subtraction, and multiplication problems (Rapp et al., 2012).

Setting of Intervention, Daily Instruction, and Mathematics Instruction

As part of this analysis, studies were reviewed to determine the setting of the interventions themselves, the daily instructional setting of
participating students, and the daily mathematics instructional setting for each participant.

**Location of intervention.** Every article in this review described studies that were conducted in secluded areas outside of the general education setting. These settings included school resource classrooms (Banda & Kubina Jr., 2010; Cihak & Foust, 2008; Cihak & Grim, 2008), self-contained classrooms (Burton et al., 2013; Hua et al., 2012; Waters & Boon, 2011; Fletcher et al., 2010), individual secluded classrooms (Rapp et al., 2012; Whitby, 2013), a student or author’s home (Rapp et al., 2012; Rockwell et al., 2011), and an autism clinic (Bouck et al., 2014).

**Location of majority of daily instruction.** The studies that did provide information regarding the setting of daily instruction indicated that 4 students received most of their daily instruction in the general education setting (Rockwell et al., 2011; Whitby, 2013), while 25 spent the majority of the day in a self-contained classroom, resource classroom, or other secluded setting (Bouck et al., 2014; Burton et al., 2013; Cihak & Foust, 2008; Cihak & Grim, 2008; Fletcher et al., 2010; Hua et al., 2012; Waters & Boon, 2011).

**Location of mathematics instruction.** The analysis also indicated that 3 students received mathematics instruction in the general education setting (Whitby, 2013), while 16 of the participating students received mathematics instruction in a self-contained classroom, resource classroom, or other secluded setting (Bouck et al., 2014; Burton et al., 2013; Hua et al., 2012; Rockwell et al., 2011; Waters & Boon, 2011). This data suggests that although an increasing number of students with ASD are being included in the general education setting, the majority of the current mathematics interventions are comprised of students who received mathematics instruction in a self-contained or other segregated setting, which may in part be a function of their overall intellectual levels. Nonetheless, these students’ access to the general education curriculum was relatively limited, preventing them from acquiring math content according to general education curricular standards.

**Mathematical Skill Targeted: Academic or Functional**

Reviewed articles were also analyzed based on the targeted skill type, either academic or functional. Academic interventions were identified as having the goal of strengthening students’ academic skills and understanding of mathematical concepts. Functional interventions were identified as having the goal of improving life skills. Academic interventions were included in seven of the reviewed articles (Banda & Kubina Jr., 2010; Bouck et al., 2014; Cihak & Foust, 2008; Fletcher et al., 2010; Rapp et al., 2012; Rockwell et al., 2011; Whitby, 2013), while functional interventions categorized four of the articles (Burton et al., 2013; Cihak & Grim, 2008; Hua et al., 2012; Waters & Boon, 2011).

A closer inspection of the seven academic interventions revealed only two interventions focused on improving mathematical word problem solving skills of students with ASD (Rockwell et al., 2011; Whitby, 2013). Three studies were implemented with the goal of teaching addition facts, specifically single-digit addition problems (Cihak & Foust, 2008; Fletcher et al., 2010) and three-digit by three-digit addition or missing addends problems (Banda & Kubina Jr., 2010). Additionally, one study focused on teaching single-digit and double-digit subtractions skills (Bouck et al., 2014), while the final study taught addition skills to two participants, addition and subtraction skills to another participant, and multiplication skills to the fourth participant (Rapp et al., 2012).

The four studies categorized as functional interventions focused on improving the money skills of students with ASD. The first of these studies taught three digit money computational subtraction problems using regrouping and touch point (Waters & Boon, 2011). The second intervention used the counting-on math technique with the next-dollar strategy to increase independent purchasing skills (Cihak & Grim, 2008). A three-step cognitive strategy (TIP) for calculating tip and total bill was taught during the third intervention (Hua et al., 2012). The final intervention used video self-modeling to teach money estimation of a given item, and the estimated amount to receive in change for
that item (Burton et al., 2013). These four interventions were all effective in teaching basic mathematical processes to improve the money skills of students with ASD. Moreover, results showed that many participants were able to apply the acquired techniques in community settings.

For the functionally driven interventions, although students’ successful implementation of critical life skills in settings beyond the classroom supports their ability to acquire, maintain, and generalize mathematical material, mathematical processes targeted were generally low level. Additionally, despite the fact that more of the reviewed studies focused on academic rather than functional skills, the academic interventions themselves also mainly targeted low-level mathematical content, including basic addition and subtraction skills. Without understanding of common advanced mathematical knowledge and processes (e.g. multiplication, division, or algebraic skills), students with ASD, particularly those less affected by intellectual disability, may not be prepared for the more rigorous academic content of the general education classroom.

**Discussion and Implications**

Given their increasing prevalence rates and more frequent placement in general education classrooms, there exists a growing need to teach general education curricular academics to students with ASD, particularly in the area of mathematics. Correspondingly, targeted attention to mathematics related IEP goals and related interventions is essential to meeting individual needs as many students with ASD require support to develop the conceptual and procedural understanding necessary to engage in mathematical computation and problem solving successfully. From this review, it is clear that research evaluating mathematics interventions for learners with ASD is preliminary and may therefore be insufficient to effectively guide teacher practice. The results of this review point to the need for future research to extend the literature base reviewed here. Future research should include investigations specific to students with ASD that are more specifically focused on the higher-order skills consistent with NCTM recommendations and that maximize students’ access to the general education curriculum as is appropriate. Moreover, evidence-based interventions found to be effective with learners with other types of disabilities (e.g., learning disability) should be systematically tested for possible usefulness for students with ASD, in accord with the quality indicators for research methodology and evidence-based practices established by the Council for Exceptional Children’s Division for Research (Odom et al., 2005).

Future single-subject design investigations should meet standard expectations for rigor by including the actual measures used in the mathematics activities, such as pre and post-test measures and generalization measures that establish a pattern of academic behavior or instances of deviation from a pattern, thereby producing more detailed information for evaluation by researchers and educators alike. Moreover, reporting how intervention fidelity is assessed is also critical. Encouragingly, among the studies reviewed herein, ten provided specific information on the extent to which interventions were implemented with fidelity (Banda & Kubina Jr., 2010; Bouck et al., 2014; Burton et al., 2013; Cihak & Foust, 2008; Cihak & Grim, 2008; Fletcher et al., 2010; Hua et al., 2012; Rockwell et al., 2011; Waters & Boon, 2011; Whitby, 2013), with nine of these studies recording a treatment integrity of at least 95% (Banda & Kubina Jr., 2010; Bouck et al., 2014; Burton et al., 2013; Cihak & Foust, 2008; Cihak & Grim, 2008; Fletcher et al., 2010; Hua et al., 2012; Rockwell et al., 2011; Whitby, 2013). However, multiple articles described treatment fidelity being calculated for only a specified number of intervention sessions, instead of being recorded for all implemented sessions (Cihak & Grim, 2008; Fletcher et al., 2010; Waters & Boon, 2011). This lack of consistent tracking of treatment fidelity is a possible limitation of the studies reviewed.

Of the eleven studies reviewed, six interventions demonstrated the skill being taught to mastery in at least three consecutive intervention or post-intervention sessions (Bouck et al., 2014; Cihak & Foust, 2008; Cihak & Grim, 2008; Fletcher et al., 2010; Rockwell et al., 2011; Waters & Boon, 2011). This mastery criterion was often recorded as 100% accuracy.
in problems completed during a session. In the remaining five studies, data showed that students improved their mathematics skills because of the intervention, but all participants did not demonstrate mastery in three consecutive sessions as is often standard in research practice (Banda & Kubina Jr., 2010; Burton et al., 2013; Hua et al., 2012; Rapp et al., 2012; Whitby, 2013). Although these latter studies recorded significant improvements as a result of the interventions, they could be strengthened by requiring students to achieve at least three consecutive sessions of mastered criterion (e.g. 100% performance on the intervention testing problems each session), which results in a greater possibility of skills being maintained over time. Additionally, nine of the reviewed articles included a post intervention assessment or follow-up to evaluate maintenance of the intervention skills and to determine the prolonged treatment effect (Bouck et al., 2014; Burton et al., 2013; Cihak & Foust, 2008; Cihak & Grim, 2008; Fletcher et al., 2010; Hua et al., 2012; Rockwell et al., 2011; Waters & Boon, 2011; Whitby, 2013). Future research should continue to focus on meeting these important standards for rigor in their design, implementation, and analysis (Horner et al., 2005).

Mathematics instruction in special education has been typified to a considerable extent by its focus on rote memorization of facts and computational skills, rather than on developing and expanding mathematical skills to be applied in authentic problem situations (Woodward & Montague, 2002). On the other hand, the NCTM has emphasized meaningful instruction for all students in order to assist them in developing understanding of mathematical content and procedures to engage in problem solving (NCTM, 2002). Few studies examined in this review referenced NCTM standards, and the majority targeted relatively basic mathematical computation skills. Because NCTM standards are for all students, future interventions for students with ASD should target such mathematics requirements. By developing interventions based on NCTM standards, students with ASD will gain stronger mathematics skills to enable them to be more successful in the general education setting. Furthermore, given that recent literature suggests nearly 40% of individuals with ASD have average to above average levels of intelligence and that ASD is becoming less strongly associated with intellectual disability than traditionally held (Croen, Grether, Hoogstrate, & Selvin, 2002), research interventions should reflect a focus on higher-order skills so that teachers can be prepared to provide access to high-quality mathematics instruction for students with ASD included in their classrooms.

Math interventions with a strong evidence base should be applied in settings serving students with ASD. Recent reviews of mathematics interventions indicate that there is empirical support for cognitive strategies including verbal strategy instruction (Montague, 2008) and visual-spatial strategies (van Garderen, 2007) as well as meta-cognitive strategies (Xin & Jitendra, 1999) for enhancing students’ mathematic performance. Such studies have been limited primarily to students with diagnosed learning disability and could be expanded to evaluate their effectiveness for students with ASD, many of who also contend with mathematics learning difficulties. Based on this review, there is emerging evidence to support cognitive and visually based strategies for learners on the autism spectrum but additional studies are needed.

**Limitations**

One specific limitation associated with this literature review is publication bias, which is a possible confound in most literature syntheses (Reichow & Volkmar, 2009). That is, there may be systematic differences between studies that were selected for inclusion and those that were excluded from the review. Although an extensive literature search was conducted utilizing multiple methods and sources, the narrow inclusionary criteria that required studies to be peer-reviewed may have created study selection bias. In addition, the recency of publication for the studies included in this review (i.e., 2008–2013) may also have narrowed its scope, though given the relatively current rise in identification of students with ASD, it is not surprising that most intervention studies are also correspondingly recent.
Conclusion

Poor post secondary outcomes for youth with ASD point to the need for a re-evaluation of the quality and quantity of academic preparation individuals with ASD receive throughout their schooling (Fleury et al., 2014). Although a substantial number of students with ASD contend with mathematical difficulties, research addressing the mathematical knowledge and skills of children and youth with ASD is in its infancy and is generally limited. Researchers should continue investigating this line of inquiry with a focus on interventions that stress conceptual knowledge and problem solving and that attend to the principles, standards, and skill areas described by the NCTM. In the interim, teachers should provide mathematics instruction from the earliest grades that is consistent with recommendations made by the NCTM and the existing research on effective mathematics interventions for students with ASD, while educational researchers expand their investigations to apply other evidence-based mathematics interventions in settings serving such students. Based on this review, emerging evidence suggests that learners with ASD can make gains in mathematics skills when targeted interventions are implemented.

References


Received: 30 January 2014
Initial Acceptance: 1 April 2014
Final Acceptance: 1 May 2014