An Investigation of the Effects of CRA Instruction and Students with Autism Spectrum Disorder

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Abstract: Students with Autism Spectrum Disorders (ASD) have unique educational needs. The concrete representational abstract (CRA) instructional sequence has been shown effective in teaching students with mathematical difficulties. The purpose of this study was to examine the effects of the CRA sequence in teaching students with ASD. A multiple baseline across behavior design was used in assessing the effects of CRA to three elementary students with ASD over four weeks of instruction. A functional relation was demonstrated between CRA and three behaviors: addition with regrouping, subtraction with regrouping, and the multiplication facts zero through five. The results and implications are discussed further.

The number of children eligible for special education services under the category of Autism Spectrum Disorder (ASD) has increased (Centers for Disease Control and Prevention, 2012). According to Rice (2009) children identified as having ASD increased 60% among males and 48% among females from the year 2002 to 2006. No Child Left Behind (2002) and the Individuals with Disabilities Educational Improvement Act (2004) make it clear that educators must provide research based instruction for students with disabilities, and that students with disabilities must show proficiency in the general education grade level standards. This includes students with ASD. The National Council of Teachers of Mathematics (NCTM, 2000), the National Mathematics Advisory Panel (NMAP, 2008), and the Common Core Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) call for in-depth instruction of whole numbers as critical for elementary aged students. In particular, the NCTM, NMAP, and Common Core Standards for Mathematics suggested elementary students must develop an understanding of the meaning of the basic operations of addition, subtraction, multiplication, and division as well as build fluency with the standard algorithms for addition, subtraction, multiplication, and division. The NCTM (2006) set focal points for mathematic instruction for prekindergarten to Grade 8 which included understanding numeric operations and their relationship to each other. The NMAP also explained that practice with conceptual understanding of whole numbers allows students to achieve automaticity of basic skills which is the fast, accurate, and effortless processing of content information. This automaticity frees up working memory for more complex aspects of problem solving. Conceptual understanding of numbers and operations demonstrated through representation of objects is an integral component of the Common Core Standards for Mathematics. Since students with disabilities, including students with ASD, are expected to make progress within the general education curriculum through the Common Core Standards for Mathematics, it is important that effective supplemental instructional methods are explored.

To date, researchers have demonstrated that a sequence of instruction referred to as concrete-representational-abstract (CRA) as
an effective means to teach and supplement mathematics instruction for students with disabilities. The CRA sequence involves three levels of instruction. The first level, the concrete, uses manipulatives to promote conceptual understanding; problems are completed through manipulation of objects. The second level, the representational level, involves completion of problems using drawing to represent numbers instead of manipulating objects. This level of instruction serves as a bridge to the final level, the abstract level in which a student uses numbers to solve problems. Instruction within each level is explicit, meaning that it includes an advance organizer, teacher demonstration, guided practice, independent practice, immediate feedback, and a post organizer.

Skills in which CRA instruction was demonstrated as a successful intervention for students with disabilities include integers, fractions, and algebra concepts (Butler, Miller, Crehan, Babbitt, & Pierce, 2003; Maccini & Hughes, 2000; Maccini & Ruhl, 2000; Witzel, Mercer, & Miller, 2003). Additionally, CRA has been shown effective in teaching basic early mathematical computation for students who struggle with learning mathematics (Flores, 2009; Harris, Miller, & Mercer, 1995; Mercer & Miller, 1992; Morin & Miller, 1998). CRA may possibly be an effective method of teaching basic computation to students with ASD (Donaldson & Zager, 2010) because research indicates that they prefer visual modes of communication and have particular difficulty understanding abstract concepts (Boutot & Myles, 2011; Ganz, Earles-Vollrath, Cook, 2011). Due to the current educational requirements (IDEA, 2004; NCLB, 2002) many students with ASD are accountable for acquiring general education standards in which knowledge of basic mathematics computational skills are key components. It is possible CRA could be a means for students with ASD to achieve general education curriculum standards.

Kroesbergen and Van Luit (2003) examined the effectiveness of mathematical interventions for students with disabilities, specifically interventions that involved explicit instruction, cognitive self-instruction, and assisted instruction. With regard to basic math computation, they found that interventions that utilize explicit instruction were more effective than cognitive self-instruction and assisted instruction for students with disabilities. Students with ASD could benefit from explicit instruction for the following reasons: (a) explicit teaching is teacher directed instruction that is organized and task oriented, (b) concepts are presented in a clear and direct manner, and (c) students respond to instruction and receive immediate feedback (Miller 2009; Peterson, Mercer, and O’Shea, 1988; Witzel et al., 2003).

Concrete Representational Abstract Instruction

Peterson et al (1988) combined explicit instruction and the use of CRA to teach place value to the tens place. Students who participated in the study received special education services for specific learning disability (SLD). There were 20 males and 4 females ranging in age from 8 to 13. A multiple baseline across subjects design was used to investigate explicit CRA instruction and student achievement regarding place value. Probes and maintenance data revealed that students mastered place value identification with an 80 percent or greater level of accuracy.

Mercer and Miller (1992) built upon Peterson et al.’s work and developed a curriculum called Strategic Math Series (SMS) utilizing explicit instruction and the CRA sequence to teach place value and basic math facts to students with disabilities. Mercer and Miller field tested SMS with 109 elementary students of whom 102 had a learning disability, five students had an emotional behavior disorder, and two were at risk. Total mean scores demonstrated that the average gain across skills was 59%.

Harris et al (1995) evaluated the effectiveness of teaching multiplication facts using explicit teaching and CRA instruction to elementary students with disabilities in the general education classrooms. Participants of this study included 13 second graders with disabilities and 99 second grade students without disabilities. The performance of students with disabilities improved; it was equivalent or slightly improved compared to that of their normally achieving peers during the phase of instruction that required demonstration of conceptual understanding of the multiplication process. With a strategy for counting ob-
jects within groups, subjects with disabilities accurately completed basic multiplication problems during independent computation and problem solving practice activities.

Morin and Miller (1998) extended the study of Harris et al. using multiplication and the SMS curriculum. The purpose of this study was to evaluate the effectiveness of teaching multiplication facts and related word problems using the CRA teaching sequence to middle school students with intellectual disabilities. A multiple baseline across subjects design was used to demonstrate experimental control and a functional relation between interventions that used the CRA sequence, and multiplication facts the students learned to solve word problems. All student improved their performance to 90% accuracy. The findings of this investigation suggest that students with intellectual disabilities were able to learn to solve multiplication facts and related word problems using the CRA teaching sequence and explicit instructional procedures.

Flores (2009) expanded on Miller and Mercer’s research using explicit instruction and CRA to teach students how to subtract with regrouping. Participants were six third-grade students who were all failing mathematics. The study used a multiple probe design to evaluate the efficacy of CRA instruction for teaching subtraction with regrouping. A functional relation was demonstrated between CRA and subtraction skills. All five students met the criterion of writing 20 digits on three consecutive 2 minute curriculum-based measures. Of the five students, four maintained their performance 4 weeks after the end of instruction.

Similar to Flores (2009), Kaffar and Miller (2011) investigated the effects of CRA instruction but included the mnemonic titled “RENAME” for subtraction with regrouping. Participants who received the instruction utilizing CRA consisted of eight students with math difficulties and three students with disabilities. Twelve students were in the control group and received instruction using a basal program. Both groups made gains in subtraction with regrouping, however, the gains made by the students who received instruction with the CRA sequence was greater. Students’ mean percentage computation scores increased from 49% to 90% for the treatment group while the comparison group’s scores increased from 66% to 72%.

All of the CRA literature reviewed included explicit instruction (an advance organizer, teacher modeling, teacher guidance, independent practice, and a post organizer) and demonstrated the CRA sequence to be effective in teaching the participants mathematics operations (Harris et al., 1995; Flores, 2009; Kaffar & Miller, 2011; Mercer & Miller, 1992; Morin & Miller, 1998; Peterson et al., 1988). Currently, studies have not been implemented to examine the effects of instruction utilizing an explicit instruction curriculum with the CRA sequence for students with ASD. This conclusion was drawn after searches that included key words: “mathematics,” “mathematics interventions,” concrete-representational-abstract,” “CRA,” “autism spectrum disorder,” “autism,” “Asperger Syndrome,” “mathematics instruction,” and “explicit mathematics instruction,” using the following databases: Psych Info, ERIC and Education Research Complete.

Mathematics instruction for students with ASD might be particularly beneficial for the following reasons. Research has shown that students with ASD prefer instruction through a visual modality ((Boutot & Myles, 2011). The use of manipulative objects and pictures provides students with a visual and concrete representation of the abstract concepts associated with numerical operations. Students with ASD perform better when instruction is structured and predictable (National Autism Center, 2009). The general education curriculum though the Common Core Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010) requires students to represent numbers and operations. Instruction using the CRA sequence provides students with explicit instruction, implemented in a structured manner that guides students through each component of a complex process of representing operations. Students with ASD represent a growing segment of learners who participate in the general education curriculum and there is currently no research regarding supplemental interventions or instructional strategies to inform classroom practice.

Therefore, the purpose of this study is to examine the effects of the CRA sequence on
the mathematics performance of students with autism spectrum disorders. The following questions guided the study. What are the effects of CRA instruction on students with ASD’s addition with regrouping performance? What are the effects of CRA instruction on students with ASD’s subtraction with regrouping performance? What are the effects of CRA instruction on students with ASD’s multiplication performance?

Method

Participants

Three male elementary students participated in the study. Larry was entering Grade 3 and Devin and Marvin were entering Grade 4 in the upcoming school year. The participants’ characteristics are included in Table 1.

The criteria for participation were: (a) eligibility for special education services under the ASD category; (b) participants received special education services in the area of mathematics; (c) participants demonstrated difficulty with mathematics achievement as assessed using the KeyMath III (Connolly, 2007); and (d) parent permission to participate in the study. Students were recruited through letters sent to all parents with their application packets to the program. Of the students for whom permission was obtained, assessments were administered to determine eligibility for participation. In addition, cognitive ability scores were obtained using the Kaufman Brief Intelligence Test 2nd edition (K-BIT, Kaufman & Kaufman, 2004).

Larry received instruction in the general education classroom for the majority of the school day during the regular school year. Larry qualified for special education services under the category of ASD according to state and federal guidelines and was originally diagnosed at age three by a pediatric neurologist. He performed significantly below average in mathematics achievement according to the KeyMath III (Connolly, 2007). Devin received instruction in the general education classroom for the majority of the school day during the regular school year. Devin qualified for special education services under category of ASD according to state and federal guidelines and was originally diagnosed at the age of two years and four months by a team of professionals at a medical school center for developmental and learning disorders. He performed significantly below the average range in mathematics achievement according to the KeyMath III. Marvin received instruction in the general education classroom for the majority of the school day during the regular school year. Marvin qualified for special education services under category of ASD according to state and federal guidelines and details regarding his original diagnosis were unavailable. He performed significantly below the average range in mathematics achievement according to the KeyMath III.

Setting

The study took place within a university-sponsored extended school year program in a rural region of the Southeastern United States. The program was one month in length and students attended the program five days a week for three hours. Instruction was provided in reading, written expression, and mathematics.
according to the students’ goals within their individualized educational programs (IEP). Mathematics instruction was provided daily for 60 minutes by a certified special education teacher who was not a researcher. The teacher had a Master’s degree and three years teaching experience in an elementary school with students with ASD. The teacher was chosen because she was knowledgeable in the intervention. Prior to beginning, the teacher demonstrated a lesson implemented with 100% accuracy for each CRA level and skill.

Materials

The research materials included instructional items such as learning sheets, manipulative items and probes for assessment purposes. The progress monitoring probes were sheets of paper with nine problems. The probes were created through the use of a website named Intervention Central in which curriculum based assessments to monitor student progress were generated. There were four different probes that were alternated for each different skill area: four addition probes (two digit by two digit) that required regrouping, four subtraction probes (two digit by two digit) that required regrouping, and four single digit multiplication probes that included the digits 0–5. For example, one sheet consisted of computational problems for addition with regrouping, one sheet consisted of computational problems for subtraction with regrouping, and one sheet consisted of computational problems for multiplication problems that include the digits 0–5. Participants completed the probes during independent morning work that lasted between five and ten minutes before daily instruction began. Once all participants completed the three probes for their morning work, instruction began.

Measures to ensure content validity for the probes were conducted for each skill area. The researchers created an assessment for each skill area (one assessment for subtraction with regrouping, one assessment for addition with regrouping, and one assessment for single digit multiplication computation with numbers 0–5). Each assessment contained 75% or more of the problems from the probes created for the participants. The researchers administered the three assessments to 20 college level students from a major four-year university. The assessments were untimed and 16 of the assessments were completed (four students chose not to volunteer). Results from the internal consistency test revealed Cronbach’s Alpha Coefficient of \( r = .72 \) for addition with regrouping, \( r = 1.00 \) for subtraction with regrouping, and \( r = .72 \) for basic multiplication computation with digits 0–5.

The CRA intervention materials included base-ten blocks made of foam and learning sheets in which problems were divided into three instructional sections: (a) model,
guided practice, and (c) independent practice. Additionally, the written computation of addition, subtraction, and multiplication subtests of the Operations domain of the KeyMath III (Connolly, 2007) and Kaufman Brief Intelligence Test II (KBIT-2; Kaufman & Kaufman, 2004) were administered to assess mathematics achievement and students’ cognitive functioning. A review of the content of the KeyMath III by the NCTM publications concluded the instrument reflected the content and process standards described in the NCTM Principles and Standards for School Mathematics (2000). Additionally, the KeyMath III scores were correlated with scores on the KeyMath Revised, Normative Update: A Diagnostic Inventory of Essential Mathematics; the Kaufman Test of Educational Achievement, Second Edition; Iowa Tests of Basic Skills instrument; the Measures of Academic Progress; and the Group Mathematics Assessment and Diagnostic Evaluation. An intercorrelation was reported for the total test and Operations domain as $r = .92$ for individuals Grades 3 through 5. Also, the internal consistency coefficient for the operation domain was reported $r = .89$ (Form A) and $r = .90$ (Form B) for individuals in Kindergarten through Grade 5. The KBIT-2 was used to gain a quick estimate of intelligence. It assesses verbal and nonverbal intelligence in people from 4 to 90 years of age. The KBIT-2 provides scores for verbal intelligence, nonverbal intelligence, and a composite of overall intelligence. Correlations of the KBIT-2 to the Wechsler Abbreviated Scale of Intelligence-III (WASI-III) and the Wechsler Abbreviated Scale of Intelligence-IV (WASI-IV) are strong at .76 and .77 respectively. Comparisons showed Full Scale and Performance scores about 4.5 points and 7 points higher on the Wechsler Abbreviated Scale of Intelligence than the corresponding KBIT-2 scales. The KBIT-2 composite of overall intelligence has an internal consistency coefficient of .93 across all ages.

**Design and Procedure**

A multiple baseline across three separate behaviors was utilized to evaluate the effects of CRA for students with ASD. Multiple baseline was used to ensure enough data were collected for the study because the program was only four weeks in length. Data were collected across the behaviors of addition with regrouping, subtraction with regrouping and multiplication facts zero to five through probes. Students completed the probes during morning work before mathematics instruction began. Probes were administered daily for each mathematical skill collecting baseline and intervention data. Probes were scored based on correct answers. During the baseline phases, no instruction was provided regarding the target mathematical skill; therefore this design was used to observe student performance with and without CRA intervention.

Upon introduction of the CRA interventions, mathematics instruction was provided on a daily basis for 5 days a week. Instruction was provided in a small group and implemented by the lead teacher of the classroom. Instructional sessions lasted approximately 20 minutes in the beginning but increased to 60 minutes (20 min per behavior) as participants reached criterion and instruction was provided for subtraction with regrouping, then single digit multiplication. All of the students began instruction together, but when one student reached criterion for the first behavior, he received instruction for the next behavior for an additional twenty minutes. Since students progressed differently, instructional groups fluctuated in size from three to one.

The criterion for phase change was six of nine problems correct because the goal of the study was to provide intensive instruction in a brief amount of time. The researchers chose six because it indicated progress, at least a fifty percent increase in performance over baseline. Instruction continued after the criterion for phase change was reached. For example the student continued instruction in addition with regrouping after instruction in subtraction with regrouping began. Additionally, the mathematics skills of addition with regrouping and subtraction with regrouping are complementary skills that can be taught at the same time. Instruction in basic multiplication facts does not require regrouping as pre-requisite and can be taught at the same time. The purpose of the summer program was remediation; therefore there was an emphasis on efficient and effective instruction. There were a total of 20 days in which instruction took place. The first week involved assessment, placement into small groups for instruction,
and baseline data collection. Data were taken for all mathematic skills (i.e., addition and subtraction with regrouping, and basic multiplication). Once three stable data points were established, instruction began for addition with regrouping. Once a student met the criterion of six problems correct, addition instruction continued; however the next phase of the study began in which the intervention for subtraction with regrouping was included. Once a student met the criterion of six problems correct for subtraction with regrouping, the third phase of the study began in which a student received instruction for multiplication facts zero through five. Students progressed differently; therefore instructional groups fluctuated in size from three to one.

**Instructional Procedures for Addition and Subtraction with Regrouping**

CRA instruction for addition and subtraction was modeled after the SMS addition and subtraction with regrouping curriculum used in previous research (Flores, 2009; Kaffar & Miller, 2011; Mercer & Miller, 1992). The concrete phase of instruction included the first three lessons. For each lesson, a sample script and learning sheets guided the teacher through the instructional sequence. During these lessons, the teacher modeled problem solving by manipulating concrete objects (i.e., base ten blocks) to solve problems. The addend or minuend was represented with the base-ten blocks and beginning in the ones place, the other addend or subtrahend was combined or taken away, trading ten ones for one ten or one ten or ten ones when regrouping was necessary. This continued for the tens and hundreds places. Then, the teacher guided students through turn-taking and together they solved problems. This was followed by independent practice in which the students completed problems without assistance. The lesson ended with a post organizer in which the lesson was reviewed.

Following the SMS curriculum procedures, representational level instruction did not begin until students completed independent practice problems with 80% accuracy. Representation level lessons proceeded using similar methods, except the problems were represented using drawings (tallies for ones, long lines for tens, and squares for hundreds). Lesson seven introduced the “RENAME” strategy (Kaffar & Miller, 2011). “RENAME” included six steps. This involved (a) read the problem, (b) examine the ones, (c) note the ones, (d) address the tens column, (e) mark the tens column, and (f) examine and note the hundreds and exit with a quick check. Abstract instruction started in lesson eight and continued until the end of the program in which RENAME helped students remember problem solving steps using numbers only.

**Instructional Procedures for Multiplication Facts 0–5**

The procedures for CRA instruction for multiplication were those associated with the SMS Multiplication Facts curriculum. They were similar to those used to teach addition and subtraction with regrouping except students used manipulative items and pictures to represent repeated addition. For example, the problem 2x2, was represented as two groups of two objects. In addition, once students mastered operations using objects and pictures, they learned “DRAW” instead of “RENAME”. The “DRAW” strategy has four steps: (a) discover the sign, (b) read the problem, (c) answer with a conceptual representation, and (d) write the answer.

**Integrity and Inter-Observer Agreement**

Data for integrity were collected throughout each of the instructional conditions within the study to ensure that research and intervention procedures were implemented correctly. Integrity checklists were completed in which two researchers observed administration of probes and instruction as it was taking place as well as digital recordings of the probe administration and instructional lessons. Integrity checks were conducted during 27% of the sessions pertaining to baseline and addition with regrouping, 30% for the sessions pertaining to baseline and subtraction with regrouping, and 60% for the sessions pertaining to baseline and multiplication facts zero through five. The checklist used to measure integrity included behaviors such as: the instructor provides students with a blank probe sheet, instructs students to complete as many problems
as possible, and does not give answers; the instructor says to students what they will be doing and why, the instructor demonstrates how to solve the problems using objects, pictures, or numbers; students and the instructor solve problems together; the instructor tells students to solve problems independently and does not offer answers; the instructor provides feedback regarding student responses; and lastly, the instructor closes with a positive statement about student’s performance in the feedback process and mentions future expectations. To calculate integrity, the researcher took the total number of agreements between two researchers and divided the number by the total number of observations; then multiplied by 100 for each condition (Poling, Method, & LeSage, 1995). The level of procedural integrity for instructional sessions was calculated as 88% for addition with regrouping condition, 97% for subtraction with regrouping condition, and 81% for multiplication facts zero through five condition. Procedural integrity was calculated at 81% for the multiplication condition because the instructor did not explain why the students were doing multiplication and did not explain future expectations consistently during multiplication lessons.

Inter-Observer Agreement was conducted for 100% of the probes administered as well as the integrity checklists. Probes were collected before each instructional lesson. Probes were scored by the instructor and later scored again by a researcher. To calculate inter-observer agreement, the total number of agreements between a researcher and the instructor was divided by the total number of agreements plus disagreements; then multiplied by the number 100. Inter-observer agreement was 100% for probes and 98% for the integrity checklists.

Social Validity

Social validity was addressed through a closed and open ended questionnaire after the study. The teacher who implemented the instruction answered questions regarding the efficacy of the intervention and recommendations for the intervention. The teacher’s feedback indicated that the SMS curriculum taught students with ASD to complete the computational problems and improved their skills. In addition, the teacher indicated the curriculum was worth the time and instructional effort, would use the intervention again, and recommended its use to others. Lastly, the teacher wrote she liked the universal approach of the curriculum when presenting math skills with concrete, representational, and abstract strategies.

Results

A multiple baseline across behaviors design was utilized to evaluate the effects of CRA for students with ASD. Data were interpreted by visual inspection and the following were noted: overlap between baseline and treatment, slope of each treatment data path, and number of data points from the beginning of treatment to criterion. Results for Larry, Devin, and Marvin are summarized in Figures 1, 2, and 3.

Larry’s baseline data were stable across all behaviors, zero correct problems on all probes. Devin’s baseline performance on addition with regrouping was more variable ranging from zero problems correct to two problems correct, however his performances on the subtraction with regrouping and multiplication probes were stable at zero problems correct. Marvin’s performances on addition and subtraction with regrouping probes were stable with zero problems correct on each, yet his performance on the multiplication probes had some variation ranging from zero problems to one problem correct.

Performance after Implementation

Larry. Larry received fifteen lessons for addition with regrouping before the Summer Program ended and completed a total of seventeen probes. He reached criterion for addition with regrouping after two probes. There was an immediate change in performance between baseline and CRA instruction no overlapping data points across the baseline and instructional phases. The instructional phase data points show an upward trend which indicates steady improvement. Larry received twelve lessons for subtraction with regrouping and completed a total of eighteen probes. He reached criterion after three probes. There
was a change in performance level; however the first data point overlapped with baseline. The instructional phase data points indicate an upward trend. Larry received eight lessons for multiplication facts and completed a total of nineteen probes. He reached criterion after three probes. There was a change in performance between baseline and CRA, no overlapping data points, and an upward trend.

**Devin.** Devin received thirteen lessons for addition with regrouping before the Summer Program ended and completed a total of 16 probes. He reached criterion for addition with regrouping after two probes. There was an immediate change in performance between baseline and CRA instruction and no overlapping data points between the baseline and instructional phases. Devin received eleven lessons for subtraction with regrouping and completed a total of seventeen probes. He reached criterion after three probes. There was a change in performance level; however the first data point overlapped with his baseline performance. Devin received nine lessons for multiplication facts and completed a total of eighteen probes. He reached criterion after one probe. There was an immediate change in performance between baseline and CRA instruction, no overlapping data points, and upward trend which indicates steady improvement.

**Marvin.** Marvin received 13 lessons for addition with regrouping before the Summer Program ended and completed a total of 15
probes. He reached criterion for addition with regrouping after one probe. There was an immediate change in performance between baseline and CRA instruction and no overlapping data points between the baseline and instructional phases. The instructional phase data points show an upward trend which indicates steady improvement. Marvin received 11 lessons for subtraction with regrouping and completed a total of 15 probes. He reached criterion after four probes. There was a change in performance level; however the first data point overlapped with baseline performance. The instructional phase data points show an upward trend. Marvin received eight lessons for multiplication facts zero to five and completed a total of 18 probes. He reached criterion after two probes. There was an immediate change in performance between baseline and CRA instruction, no overlapping data points, and upward trend which indicates steady improvement.

**Effect size**

Tau-U was calculated for each student; this form of analysis combined non-overlapping data points between phases with trend within the intervention phases while accounting for any trend within baseline (Parker, Vannest, Davis, & Sauber, 2011). For Larry, there were no significant trends within baseline phases. In comparing baseline and intervention phases for addition with regrouping, a strong effect was indicated (Tau-U = 1). In comparing Larry’s baseline and intervention data for subtraction with regrouping, a strong effect was indi-
cated (Tau-U = .90). The comparison of Larry’s baseline and intervention phases for multiplication indicated a strong effect (Tau-U = 1). Overall, the intervention had a strong effect across all phases (Tau-U = .97).

For Devin, there were no significant trends within baseline phases. In comparing baseline and intervention phases for addition with regrouping, a strong effect was indicated (Tau-U = 1). In comparing Devin’s baseline and intervention data for subtraction with regrouping, a strong effect was indicated (Tau-U = .90). The comparison of Devin’s baseline and intervention phases for multiplication indicated a strong effect (Tau-U = 1). Overall, the intervention had a strong effect across all phases (Tau-U = .97).

For Marvin, there were no significant trends within baseline phases. In comparing baseline and intervention phases for addition with regrouping, a strong effect was indicated (Tau-U = 1). In comparing Marvin’s baseline and intervention data for subtraction with regrouping, a strong effect was indicated (Tau-U = .90). The comparison of Marvin’s baseline and intervention phases for multiplication indicated a strong effect (Tau-U = 1). Overall, the intervention had a strong effect across all phases (Tau-U = .97).

Discussion
In the current study, the authors investigated the effectiveness of CRA for teaching addition and subtraction with regrouping and multiplication facts zero to five to students with ASD.
A functional relation was demonstrated between the CRA instruction and the behaviors of addition and subtraction with regrouping and multiplication facts zero to five with three participants. The students participated in each level of instruction without modifications, and moved through the phases of instruction with relative ease. All three students demonstrated steady progress across the three skill areas. The students’ positive outcomes may have been related to the visual nature of instruction. This involved watching the teacher move objects to demonstrate the mathematical concept of addition then allowing the students to manipulate objects themselves demonstrating their understanding. For example, Marvin improved after one day of instruction for addition with regrouping. Marvin demonstrated what addition with regrouping looked like with objects and received immediate feedback. Therefore, when he solved problems using numbers only, his previous demonstrations of addition with regrouping at the conceptual level using manipulative items and pictures may have made the task easier. Perhaps Marvin’s sudden increase in performance was related to the correction an error pattern or conceptual misunderstanding, but CRA instruction corrected his misconceptions or lack of procedural understanding. This might also explain the sudden changes observed for Devin in the multiplication condition. Based on background information and their individualized education programs, the students had received instruction in these areas during their regular school year using general education curriculum materials. Perhaps instructional design and the use of the CRA sequence were more conducive to learning and consistent with research related to students with ASD’s preference for visual input (Ganz et al., 2011). Also, Instruction was explicit and followed predictable patterns of demonstration, guided practice, and immediate feedback, consistent with instructional standards for students with ASD (National Autism Center, 2009).

Consistent with previous research (Harris et al., 1995; Flores, 2009; Kaffar & Miller, 2011; Mercer & Miller, 1992) the effects of the CRA instructional sequence resulted in mathematical computation gains. The students’ pretests indicate poor performance regarding addition and subtrac-

tion with regrouping as well as basic multiplication. The CRA instructional sequence provided students with clear expectations and the scaffolding regarding the conceptual meaning of operations to support the procedural knowledge needed to complete the problems. Perhaps the students made gains because instruction involved focus on conceptual knowledge through manipulation of objects and immediate feedback from the teacher. This is consistent with Donaldson and Zager’s (2010) argument that CRA’s explicit nature and scaffolding from concrete to abstract may benefit students with ASD. The CRA sequence provided explicit instruction, appealing to students’ visual preferences (Boutot & Myles, 2011; Ganz et al, 2011).

The skills gained by the students in this study are consistent with learning standards within the Common Core Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). The students demonstrated increased conceptual knowledge of operations, showing knowledge of place value and its relation to operations through regrouping. In addition, students demonstrated knowledge of multiplication and its relation to addition by repeatedly adding groups. This is significant because these skills will allow for progress within the general education curriculum with more complex concepts.

This study extends the current literature (Flores 2009; Harris et al., 1995; Kaffar & Miller, 2011; Morin & Miller, 1998) by investigating the effects of the CRA instructional sequence and mathematics achievement for students with ASD. This is significant in that it provides an initial investigation of interventions for students with ASD. With the introduction of the Common Core Standards for Mathematics (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010), it is important to continue to conduct research regarding effective interventions and to also include students with disabilities who will participate in general education instruction such as students with ASD.

Limitations and Future Research

The research design presents a limitation to the present study because the CRA instructional sequence was not compared with an-
other mathematics program. Therefore, there may be other instructional programs or interventions as or more effective. Another limitation of this study is that social validity data were not collected directly from the students who participated in the study; only the teacher provided social validity data. In future studies, data from the students must be collected and reported. Lastly, generalizability of the results across settings is a limitation because the setting is outside the general education classroom and involves small group instruction. It is unclear if the same effects would be replicated in a larger group, inclusive setting. Also, due to the limited time frame of the program, maintenance data were not collected. Therefore, the long term effects of the instructional intervention are unclear.

Future research should be conducted in order to extend and replicate findings related to CRA and increased computation performance for students with ASD. Since there is little research for this population related to mathematics interventions, it is important to investigate the efficacy and efficiency of mathematics interventions such as CRA. Although CRA has been shown to be effective for students with high incidence disabilities, there may be particular aspects of CRA or other interventions that are effective for students with ASD who participate in the general education curriculum. Another area of future research may be an examination of the long term effects of CRA instruction for students with ASD. Researchers should investigate the effects of CRA in an inclusive setting for students with ASD and with larger groups of participants. In addition, research should be conducted to compare the effects of CRA with other mathematical programs that provide instruction for students with ASD.

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